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Level 4

Module Code: M4X00391

Mathematics (Test 2)

2 hours

Q1=[20/35]

Q2=[28/33]

75%

Q3=[27/32]

Question 1

a) $A=\begin{bmatrix}2&0&3\\1&8&3\\0&2&0\end{bmatrix}$ $B=\begin{bmatrix}3&0&2\\2&5&0\\1&3&6\end{bmatrix}$ $C=\begin{bmatrix}1&4\\2&-5\\3&1\end{bmatrix}$ $D=\begin{bmatrix}2&4\\-3&8\end{bmatrix}$

i) ~~|A|~~ $|A|=\begin{bmatrix}2&0&3\\1&8&3\\0&2&0\end{bmatrix}\begin{matrix}2&0\\1&8\\0&2\end{matrix}$

$0+6+0=6$

$6-6=0$

$0+0+6=6$

~~|A|~~

$0-6$

$8\times0-3\times2$

$2\begin{bmatrix}8&3\\2&0\end{bmatrix}-0\begin{bmatrix}1&3\\0&0\end{bmatrix}+3\begin{bmatrix}1&8\\0&2\end{bmatrix}$

$0-0$

$2-0$

$2(-6)-0+3(2)$

$-12+6=-6$ ✓ [4/4]

$|A|=-6$

ii) $|C|$ $\begin{bmatrix}1&4\\2&-5\\3&1\end{bmatrix}\begin{matrix}1&4\\2&-5\\3&1\end{matrix}$

$-15+8=-7$

$=0$

Myp $-15+8=-7$

$|C|=0$

cannot solve ✓ [1/1]

iii) $|D| = \begin{bmatrix} 2 & 4 \\ -3 & 8 \end{bmatrix}$ $2 \times 8 - 4 \times -3 = 28$

$|D| = 28$ [2/2]

iv) $AB = \underset{3\times3}{A} = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ 0 & 2 & 0 \end{bmatrix}$ $\underset{3\times3}{B} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 1 & 3 & 6 \end{bmatrix}$

$2\times3+0\times2+3\times1 = 9$
$2\times0+0\times5+3\times3 = 9$
$2\times2+0\times0+3\times6 = 22$
$1\times3+8\times2+3\times1 = 22$
$1\times0+8\times5+3\times3 = 49$
$1\times2+8\times0+3\times6 = 20$
$0\times3+2\times2+0\times1 = 4$
$0\times0+2\times5+0\times3 = 10$
$0\times2+2\times0+0\times6 = 0$

$AB = \underset{3\times3}{\begin{bmatrix} 9 & 9 & 22 \\ 22 & 49 & 20 \\ 4 & 10 & 0 \end{bmatrix}}$ [4/4]

vi) BC

$$B = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 1 & 3 & 6 \end{bmatrix}_{3\times3} \qquad C = \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & 1 \end{bmatrix}_{3\times2}$$

$3\times1 + 0\times2 + 2\times3 = 9$

$3\times4 + 0\times-5 + 2\times1 = 14$

$2\times1 + 5\times2 + 0\times3 = 12$

$2\times4 + 5\times-5 + 0\times1 = -17$

$1\times1 + 3\times2 + 6\times3 = 25$

$1\times4 + 3\times-5 + 6\times1 = -5$

$$\begin{bmatrix} 9 & 14 \\ 12 & -17 \\ 25 & -5 \end{bmatrix}_{3\times2} = BC$$ [4/4]

vi) $DC = \begin{bmatrix} 2 & 4 \\ -3 & 8 \end{bmatrix}_{2\times2} C = \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & 1 \end{bmatrix}_{3\times2}$

= cannot solve. [1/1]

vii) $AD = A \begin{bmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ 0 & 2 & 0 \end{bmatrix}_{3\times3} D = \begin{bmatrix} 2 & 4 \\ -3 & 8 \end{bmatrix}_{2\times2}$

= cannot solve [1/1]

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$0 + 70 + 8 = 78$

b) $A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 4 & 7 \\ 0 & 5 & 2 \end{bmatrix} \begin{matrix} 2 & 4 \\ 1 & 4 \\ 0 & 5 \end{matrix}$

$5 - 78 \neq |A| - 73$ ✗

$0 + 0 + 5 = 5$

c) use Cramers rule to solve

$3x - 2y + z = 2$

$x - 3y + 2z = 1$

$2x + y - 3z = -5$

$\begin{pmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -5 \end{pmatrix}$ ✗ ✓

$|A| = 3\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$

$3 \times 7 - 2(-1) + 1(-5)$

$|A| = 18$ ✗ $-15 + 1 + 6 = -5$ [3/8]

$A_1 = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 2 \\ -5 & 1 & 3 \end{bmatrix} \begin{matrix} 2 & 2 \\ 1 & 3 \\ -5 & 1 \end{matrix}$

$18 + -20 + 1 = -1$

$-1 - 5 = 4 = |A_1|$

$A_2 = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & -5 & 3 \end{pmatrix}$ $\begin{matrix} 2 \\ 3 \\ 1 \\ 2 \end{matrix}$ $\begin{matrix} 2 \\ 2 \\ 1 \\ -5 \end{matrix}$

$2 + -30 + 6 = -22$

$9 + 8 + -5 = 12$

$12 - -22 = 34 = |A_2|$

correct method
error carried over

$A_3 = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & -5 \end{pmatrix}$ $\begin{matrix} 3 \\ 1 \\ 2 \end{matrix}$ $\begin{matrix} 2 \\ 3 \\ 1 \end{matrix}$

$12 + 3 + -10 = 5$

$-45 + 4 + 2 = -37$

$-39 - 5 = -44 = |A_3|$

correct method
error carried ove

$x = \frac{4}{18} = 0.2$ $\quad y = \frac{34}{18} = 1.8^{\circ}$ $\quad z = \frac{-44}{18} = -2.4$

Q2

a) $z_1 = 2 + 5j \quad z_2 = 3 + 2j \quad z_3 = 4 + 3j$

i) $z_1 + z_2 \quad (2 + 5j) + (3 + 2j) = (5 + 7j)$ [1/1]

ii) $3(3 + 2j) + 2(4 + 3j) = (9 + 6j) + (8 + 6j) = (17 + 12j)$ [3/3]

iii) $z_1^2 + z_2$

$(2 + 5j)^2 + 3 + 2j$

$(2 + 5j)(2 + 5j) + (3 + 2j)$

$4 + 10j + 10j + 25j^2 \quad (-1) \quad + (3 + 2j)$

$4 + 20j - 25 \quad + (3 + 2j)$

$(-2 + 20j) + (3 + 2j) = (1 + 22j)$ [2/3]

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ii) $(2+5j)(3+2j)$ 2122

$= 6 + 4j + 15j + 10j^2$

$= 6 + 19j + 10(-1)$

$= -10 + 6 + 19j$

$= -4 + 19j$ [3/3]

v) $\frac{z_1}{z_2} = \frac{2+5j}{3+2j} \times \left(\frac{3-2j}{3-2j}\right) = \frac{9 - 4j + 15j - 10j^2}{9 - 6j + 6j - 4j^2}$

$= \frac{9 + 11j + 10}{9 + 4} = \frac{19}{13} + \frac{19 + 11j}{13}$

$= \frac{19}{13} + \frac{11}{13}j$ ✓ [4/5]

b) $5x^2 - 8x + 5 = 0$

if $ax^2 + bx + c = 0$

then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(5)}}{2 \times 5}$ ✓ [1/1]

$x = \frac{-(-8) \pm \sqrt{-64 - 100}}{10}$

$x = \frac{-(-8) \pm \sqrt{-164}}{10}$

$$x = \frac{-(-8) \pm \sqrt{\quad} \; 12 \cdot 8j}{10}$$

$$x = \frac{-(-8)}{10} \pm \frac{12 \cdot 8}{10} j \quad [1/2]$$

$x = 0 \cdot 8 + 1 \cdot 28j$ [1/1] $\quad x = 0 \cdot 8 - 1 \cdot 28j$ [1/1]

$r = \sqrt{0 \cdot 8^2 + 1 \cdot 28^2} = 1 \cdot 51$; $\quad r = 1 \cdot 51$

$\theta = \tan^{-1}\left(\frac{1 \cdot 28}{0 \cdot 8}\right) = 57 \cdot 99°$; $\quad \theta = 57 \cdot 99$ [1/2]

$\theta = 180 - 57 \cdot 99 = 122 \cdot 01$ [2/3] ; $\quad 360 - 57 \cdot 99 = 302 \cdot 01$

$0 \cdot 8(\cos(122 \cdot 01) + j \sin(122 \cdot 01))$; $\quad 0 \cdot 8(\cos(302 \cdot 01) + j \sin(302 \cdot 01))$

$= 0 \cdot 29\,25$ $\quad x = -0 \cdot 25$

c) Use De Moivres Theorem

$(3+4j)^4$ [8/8]

$r = \sqrt{3^2 + 4^2} = 5$

$\tan^{-1}\left(\frac{4}{3}\right) = 53 \cdot 13$

$(5)^4(\cos 4 \times 53 \cdot 13 + j \sin 4 \times 53 \cdot 13)$

$625(\cos 212 \cdot 52 + j \sin 212 \cdot 52)$

$= -527 \cdot 1 - 336j$

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Q3

a) Integrate.

i) $\int x(x^2+3x)\,dx$

$= \int x^3 + 3x^2\,dx \quad = \int x^3 + 3\int x^2\,dx$

$= \frac{x^4}{4} + \frac{3x^3}{3} + C$ [4/4]

ii) $\int_0^2 (3x^2+2x-5)\,dx$ $\int ((x+2)(2x+1) - \sin 4x)\,dx$

$\int 2x^2 + x + 4x + 2 - \sin(4x)\,dx$

$\int 2x^2 + 5x + 2 - \sin 4x\,dx$

$= \frac{2x^3}{3} + \frac{5x^2}{2} + 2x + \frac{1}{4}\cos 4x + C$ [4/4]

iii) $\int_0^2 (3x^2+2x-5)\,dx = \frac{3x^3}{3} + \frac{2x^2}{2} - 5x$

$(2^3 + 2^2 - 5(2)) - 0 = 2$ [4/4]

iv) $\int_0^1 (5x^4 + 3e^x)\,dx = \frac{5x^5}{5} + \frac{3}{x}e^x$

$5 + 8.16 - 3 = 10.16$ [2/4]

b) let $u = 3x^2 + 4$ $\frac{du}{dx} = 3x$ $\frac{du}{3x} = dx$

$\int 6x(3x^2+4)^5\,dx$

$\int 6x(u)^5 \frac{du}{3x}$ [3/6]

$= \int 2(u)^5 \frac{du}{x}$

$= 2\int u^5 \frac{du}{x}$

$= 2(u^6) + c$

$= 2(3x^2+4)^6 + c$

c) Integration by parts $\int u\,dv = uv - \int v\,du$

$\int x \cdot \sin x \, dx$

$u = x$

$\frac{du}{dx} = 1 \quad du = dx$

$dv = \sin x \, dx$ [10/10]

$dv = \int \sin x \, dx$

$v = -\cos x$

$x \cdot -\cos x - \int -\cos x \, dx$

$= x \cdot -\cos x + \sin x + C$

$= -x \cos x + \sin x + c$

END